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Synchronizing the fractional-order Genesio-Tesi chaotic system using Adaptive control

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Abstract— In this paper, we address chaos control and synchronization problems of a chaotic system. Based on the Lyapunov stability theory and adaptive theory, the adaptive control law is derived such that the trajectory of the chaotic system with unknown parameters can be globally stabilized to an unstable equilibrium point of the uncontrolled system. Meanwhile, an adaptive control approach is presented to the synchronization between two identical chaotic systems. It is shown in detail that the chaos control and synchronization phenomena of this 3D chaotic system can be realized by designing suitable adaptive control laws. In addition, numerical examples are presented to illustrate the feasibility and effectiveness of the theoretical analysis

Index Terms— Synchronizing, Genesio-Tesi chaotic system , fractional-order system , Adaptive control

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1 Introduction

Chaotic behavior of dynamical systems can be utilized in many real-world applications such as circuit [1], mathematics [2], power systems [3], medicine [4], biology [5], chemical reactors [6], and so on. Therefore chaos is one of the most interesting topics which have attracted many researchers in all scientific fields. Fractional calculus can date from three hundred years ago; however, its applications to physics and engineering have just started in the recent decades [7–10]. It was found that many systems in interdisciplinary fields can be elegantly modeled with the help of the fractional derivatives such as the nonlinear oscillation of earthquakes [11], viscoelastic systems [12], diffusion waves [13], electromagnetism [14], mechanics [15], and so on. Recently, the control and synchronization of the fractional-order chaotic systems has been one of the most interesting topics, and many researchers have made great contributions. For example, in [16], chaos synchronization of the fractional-order unified systems is theoretically and numerically studied using the one-way coupling method. In [17], chaos synchronizations of two uncoupled fractional-order chaotic modified Duffing systems are obtained. A controller based on active sliding mode theory to synchronize fractional-order chaotic systems in master–slave structure was proposed in [18]. In [19], the fractional Routh–Hurwitz conditions are used to control chaos in the fractional-order modified autonomous vanderPol–Duffing system to its equilibria. In [20], an intelligent robust fractional surface sliding mode control for a nonlinear system is studied. In [21], a new fractional-order hyperchaotic system is proposed, and using the pole placement technique, a nonlinear state observer is designed to synchronize a class of nonlinear fractional-order systems. In [22], it concerns the existence of mild solutions for semilinear fractional evolution equations and optimal controls in the α-norm. Three schemes are designed to achieve chaos synchronization of the fractional-order hyperchaotic system in [23]. In [24], the classical control theory to a fractional diffusion equation was applied in a bounded domain. In [25], the authors analyzed the chaotic behavior of the fractional-order modified coupled dynamos system concretely, and provided the conditions suppressing chaos to unstable equilibrium points, then used the feedback control method to control chaos in the fractional-order modified coupled dynamos system. In [26], the function projective synchronization between fractional-order chaotic systems was investigated. In [27], a simple but efficient method to control fractional-order chaotic systems is proposed using the generalized T-S fuzzy model and adaptive adjustment mechanism.

2 Fractional-Order Derivative

Definition

The differintegral operator, represented by ${}_0D_t^q$, is a combined differentiation-integration operator commonly used in fractional calculus and general calculus operator, including fractional-order and integer is defined as:

$$D_t^q = \begin{cases} \dfrac{d^q}{dt} & q > 0 \\ 1 & q = 0 \\ \int_0^t \left(d\tau\right)^{-q} & q < 0 \end{cases} \quad (1)$$

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There are several definitions of fractional derivatives [28]. The best-known one is the Riemann-Liouvile definition, which is given by

$$\frac{d^q f(t)}{dt^q} = \frac{1}{\Gamma}\frac{d^n}{dt^n}\int_o^t \frac{f(\tau)}{(t-\tau)^{q-n+1}}d\tau \quad (2)$$

Where n is an integer such that $n-1<q<n$, $\Gamma(0)$ is the Gamma function. The geometric and physical interpretation of the fractional derivatives was given as follows

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt \quad (3)$$

The Laplace transform of the Riemann-Liouville fractional derivative is

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = s^q L\{f(t)\} - \sum_{k=o}^{n-1} s^k \left[\frac{d^{q-1-k} f(t)}{dt^{q-1-k}}\right] \quad (4)$$

Where, L means Laplace transform, and s is a complex variable. Upon considering the initial conditions to zero, this formula reduces to

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = s^q L\{f(t)\} \quad (5)$$

The Caputo fractional derivative of order α of a continuous function $f: R^+ \to R$ is defined as follows

$$\frac{d^q f(t)}{dt^q} = \begin{cases} \dfrac{1}{\Gamma(m-q)}\displaystyle\int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q-m+1}} & m-1<q<m \\ \dfrac{d^m}{dt^m} f(t) & q=m \end{cases} \quad (6)$$

Thus, the fractional integral operator of order α can be represented by the transfer function $H(s) = \frac{1}{s^q}$ in the frequency domain.

The standard definition of fractional-order calculus does not allow direct implementation of the fractional operators in time-domain simulations. An efficient method to circumvent this problem is to approximate fractional operators by using standard integer-order operators. In Ref. [29], an effective algorithm is developed to approximate fractional-order transfer functions, which has been adopted in [16] and has sufficient accuracy for time-domain implementations. In Table 1 of Ref [17], approximations for $1/s^q$ with α from 0.1 to 0.9 in step 0.1 were given with errors of approximately 2 dB. We will use the $1/s^{0.95}$ approximation formula [30] in the following simulation examples.

$$\frac{1}{s^{.95}} \approx \frac{1.2831s^2 + 18.6004s + 2.0833}{1.2831s^3 + 18.4738s^2 + 2.6574s + 0.003} \quad (7)$$

In the simulation of this paper, we use approximation method to solve the fractional-order differential equations.

3 ADAPTIVE CONTROL OF THE CHAOTIC SYSTEM

The Genesio–Tesi chaotic system, proposed by Genesio and Tesi is one of paradigms of chaos since it captures many features of chaotic systems. It includes a simple square part and three simple ordinary differential equations that depend on three positive real parameters .The dynamic equation of the system is as follows:

$$\begin{aligned} D^q x_1 &= x_2; \\ D^q x_2 &= x_3; \quad (8) \\ D^q x_3 &= -cx_1 - bx_2 - ax_3 + x_1^2; \end{aligned}$$

Where x1; x2; x3 are state variables, and a, b and c are the positive real constants satisfying ab<c.

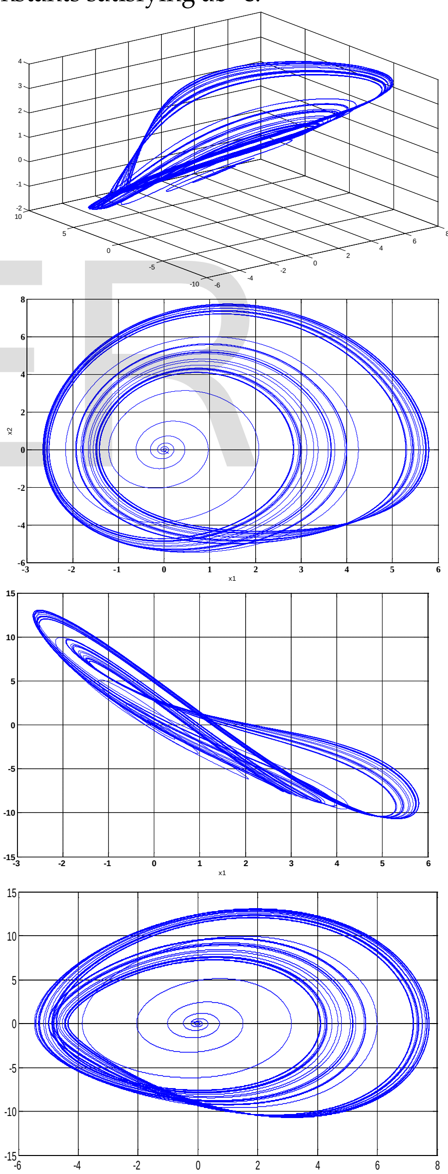

Figure 1. Chaotic attractor of system (8)

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In this section, synchronization between two identical new chaotic systems is achieved based on the Lyapunov stability theory and the adaptive control theory. Suppose the drive and response systems are given respectively as follows

$$D^q x_1 = x_2;$$
$$D^q x_2 = x_3; \qquad (9)$$
$$D^q x_3 = -cx_1 - bx_2 - ax_3 + x_1^2;$$

And

$$D^q y_1 = y_2 + u_1;$$
$$D^q y_2 = y_3 + u_2; \qquad (10)$$
$$D^q y_3 = -cy_1 - by_2 - ay_3 + y_1^2 + u_3;$$

Define:

$$e_1 = y_1 - x_1$$
$$e_2 = y_2 - x_2 \quad (11)$$
$$e_3 = y_3 - x_3$$

The error dynamic is described by

$$D^q e_1 = e_2 + u_1;$$
$$D^q e_2 = e_3 + u_2; \qquad (12)$$
$$D^q e_3 = -ce_1 - be_2 - ae_3 + y_1^2 - x_1^2 + u_3;$$

Then, the time derivative of the Lyapunov function becomes

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \quad (13)$$

Calculating the time derivative of the Lyapunov function (13) along the trajectory of system (12) yields

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3$$

$$\dot{V} = e_1 d^{1-q}(d^q e_1) + e_2 d^{1-q}(d^q e_2) + e_3 d^{1-q}(d^q e_3) \quad (14)$$

$$\dot{V} = e_1 d^{1-q}(e_2 + u_1) + e_2 d^{1-q}(e_3 + u_2)$$
$$+ e_3 d^{1-q}(-ce_1 - be_2 - ae_3 + y_1^2 - x_1^2 + u_3)$$

If we choose the following adaptive control law

$$u_1 = -e_2 - d^{q-1} A_1 e_1$$
$$u_2 = -e_3 - d^{q-1} A_2 e_2 \qquad (15)$$
$$u_3 = ce_1 + be_2 + ae_3 - y_1^2 + x_1^2 - d^{q-1} A_3 e_3$$

Then, the time derivative of the Lyapunov function becomes

$$\dot{V} = -A_1 * e1^2 - A_2 * e_2^2 - A_3 * e_3^2 < 0 \quad (16)$$

4 Simulation Result

For numerical simulation, time step size 0.001. The initial values of the drive system (8) and the response system (9) are taken as x1(0) = 1, x2 (0) = 1 and x3(0) = 1 and y1(0) = 0.5, y2(0) = 0.5, y3(0) = 0.5, respectively. Order of chaotic systems is (q=0.95) and a=1.2;b=2.92;c=6;
. Are chosen as (A1, A2, A3) = (10,10 ,10) respectively

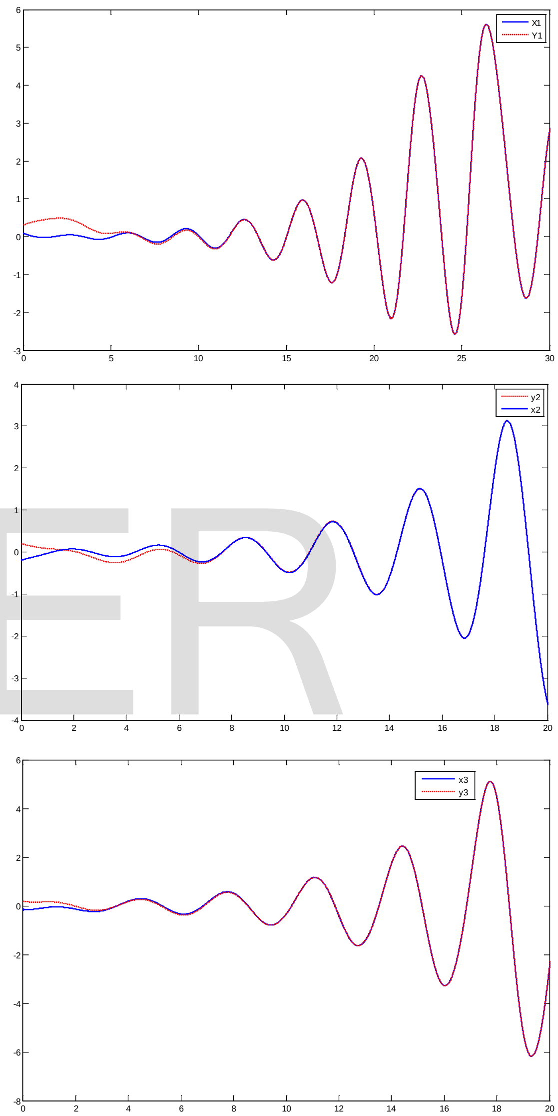

Figure 2: Synchronization performance of fractional-order

5 Conclusions

We investigate chaos synchronization of a fractional order chaotic system via adaptive control method. Based on the Lyapunov stability theory and the adaptive control theory, this fractional order chaotic system is suppressed to its unstable equilibrium. In addition, an adaptive control law and a parameter estimation update law are proposed to achieve synchronization between two identical fractional order chaotic systems.

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